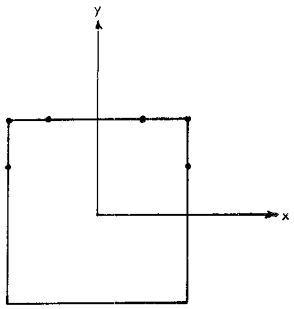

Fig. 1. The square guide with the six chosen points.

for TM wave modes, and

$$\alpha = \left(R_s/2Z_0\zeta \int_S |\psi|^2 dS\right)\left[(\zeta/k)^2 \oint_C F(r_c,\theta) r_c\, d\theta + (f_c/f)^2 \phi_C |\psi(r_c,\theta)|^2 r_c\, d\theta\right] \quad (10)$$

for TE wave modes. The integrations in (9) and (10) can be performed numerically.

II. Numerical Example

To demonstrate the validity of the point-matching method for determination of the field distribution, power transfer, and the attenuation constant, it is assumed that an electromagnetic wave is propagating inside a square waveguide in the TE_{10} mode. The guide has a width of $2a$ and is placed with its center at the origin of a rectangular coordinate system as illustrated in Fig. 1. Since the longitudinal field component H_z is symmetrical with respect to the x-axis for TE_{10}, the sine terms in (1) are omitted. The cutoff wave number calculated by using six points only on the upper half of the guide's cross-sectional contour is 1.5716, compared with the exact value of 1.5708. The expansion coefficients were determined in terms of the coefficient A_1, which is equal to a preassigned value of unity. The resulting wave function is therefore expressed in the following form:

$$\psi = H_z = \sum_{n=1}^{3} (-1)^{n+1} J_{2n-1}(kr) \cos(2n-1)\theta \quad (11)$$

with three-place accuracy. The disappearance of the even terms in (11) is not surprising, because H_z for TE_{10} is antisymmetric with respect to the y-axis. Equation (11) is a good approximation when compared with the exact solution

$$\psi = 0.5 \sin x = \sum_{n=1}^{\infty} (-1)^{n+1} J_{2n-1}(x) \cos(2n-1)\theta \quad (12)$$

since $J_7(kr)/J_1(kr) < 0.001$ for the largest value of r which is $\sqrt{2}\, a$.

The power transported in the waveguide was calculated numerically using (3) and

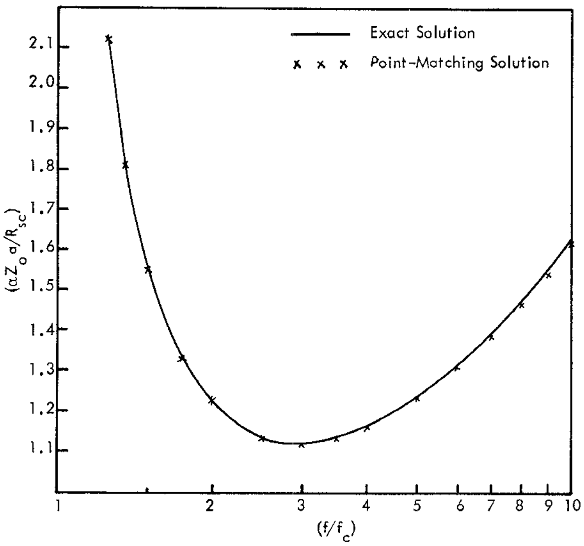

Fig. 2. The attenuation constants of the square waveguide.

(11), and the result was 0.1256, which is excellent when compared with the exact value of 0.1250.

The attenuation constant is obtained by substituting the approximate wave function (11) into (10) and performing numerical integrations. It is

$$(\alpha_p a Z_0/R_{sc}) = (0.994/2\zeta)\sqrt{f/f_c}\,[1 + 2.014(f_c/f)^2] \quad (13)$$

where $R_{sc} = R_s\sqrt{f_c/f}$, the surface resistance at cutoff frequency, and α_p denotes the attenuation constant of the point-matching solution. The exact attenuation constant α_e for a square guide of $2a$ is given by[2]

$$(\alpha_e a Z_0/R_{sc}) = (1/2\zeta)\sqrt{f/f_c}\,[1 + 2(f_c/f)^2]. \quad (14)$$

The comparison of these two attenuation constants is shown in Fig. 2 over the frequency range of $(f/f_c) = 1$ through 10. The validity of the attenuation constant calculated by the point matching solutions is then verified.

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On Transverse Electromagnetic Wave Propagation in a Cylindrically Stratified Magnetoplasma

This communication is concerned with a second-order ordinary differential equation arising in the theory of electromagnetic waves in a cylindrically stratified, axially magnetized plasma. The equation describes transverse propagation when the wave's magnetic field has only an axial component.

Galejs [1] and Yeh and Rusch [2], [3] have studied the transverse propagation of electromagnetic waves in a cylindrically stratified, axially magnetized plasma. It was found that in a continuously varying plasma, the fields are described by two uncoupled wave equations. Thus, the total field can be expressed as the sum of two partial fields which propagate independently. These may be called E-parallel and H-parallel fields, with the electric and magnetic vectors, respectively, having only axial components.

The differential equation describing E-parallel fields is unaffected by the static magnetic field [3]. The equation describing H-parallel fields was derived [1], [3], and its normal form will be considered here.

Cylindrical polar coordinates (r, ϕ, z) are used, with the static magnetic field in the z direction. The inverse permittivity tensor (ϵ^{-1}) in the magnetoplasma is given by [4], [5]

$$\epsilon_0(\epsilon^{-1}) = \begin{pmatrix} M & -iK & 0 \\ iK & M & 0 \\ 0 & 0 & \epsilon_0/\epsilon'' \end{pmatrix} \quad (1)$$

where ϵ_0 is the permittivity of free space. The quantities M, K, and ϵ'' have been defined by Wait [4], [5]. A time factor $e^{i\omega t}$ is taken where ω is the angular frequency of the fields and t is the time. The plasma is taken to be cylindrically stratified with M, K, and ϵ'' depending on r. The permeability has the free space value μ_0.

For propagation transverse to the imposed field, the fields are independent of z. Consider the case in which the magnetic field of the wave has only a z component H. The variables can be separated by writing

$$H = \sum_{n=-\infty}^{\infty} a_n [rM(r)]^{-1/2} f_n(r) e^{-in\phi} \quad (2)$$

where n is an integer and a_n is independent of the coordinates. It is found from Maxwell's equations that $f_n(r)$ satisfies

$$\frac{d^2 f_n}{dr^2} + I(r) f_n = 0 \quad (3)$$

where

$$I(r) = \frac{k_0^2}{M} + \frac{1}{4}\left(\frac{1}{M}\frac{dM}{dr}\right)^2 - \frac{1}{2M}\frac{d^2M}{dr^2} - \frac{1}{2rM}\frac{dM}{dr} - \frac{4n^2-1}{4r^2} - \frac{n}{rM}\frac{dK}{dr} \quad (4)$$

in which $k_0^2 = \omega^2 \mu_0 \epsilon_0$.

Equations (3) and (4) may be compared with equations describing propagation in a planar stratified magnetoplasma [6]. Comments similar to those made previously [6] will now apply in the cylindrical case.

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When there is no imposed magnetic field, $K=0$ and M^{-1} is equal to the relative permittivity of the medium. When $K=0$, (3) and (4) reduce to the correct equations for the no-field case [7].

It is seen further that if K is independent of r, whether it is zero or not, (3) and (4) reduce to the forms appropriate to the propagation of H-parallel fields in a cylindrically stratified unmagnetized medium with relative permittivity $1/M$. This holds even though a nonzero value of K means that there is an imposed field present.

Thus, any known solution to (3) for the no-field case [7] can be used to obtain a solution when the plasma is magnetized. If the relative permittivity in the no-field case is $\epsilon_r(r)$, then the same wave equation will apply in a magnetized medium with electrical parameters M and K, where $M=1/\epsilon_r(r)$ and K does not vary with r.

Furthermore, if $K(r)$ has the form

$$K(r) = a + br^2 \qquad (5)$$

where a and b are independent of r, then (4) becomes

$$I(r) = \frac{k_0^2 - 2nb}{M} + \frac{1}{4}\left(\frac{1}{M}\frac{dM}{dr}\right)^2 - \frac{1}{2M}\frac{d^2M}{dr^2} - \frac{1}{2rM}\frac{dM}{dr} - \frac{4n^2-1}{4r^2}. \qquad (6)$$

This has the form appropriate to propagation in an unmagnetized medium of relative permittivity $(1-2nbk_0^{-2})/M$. Thus, once again, a known solution for the unmagnetized case can be used to obtain a solution for the axially magnetized medium. In the magnetized case $M=(1-2nbk_0^{-2})/\epsilon_r(r)$, where $\epsilon_r(r)$ is the relative permittivity in the no-field case. The expression for M so obtained will depend in general on the mode number n. To eliminate this dependence, appropriate choices must be made of the arbitrary parameters involved in the expression for $\epsilon_r(r)$ in the unmagnetized medium.

Now consider H-parallel fields for the particular case in which $n=0$, so that the fields are independent of ϕ as well as of z. The function $g(r)$ is defined by writing $E_\phi = r^{-1/2}g(r)$, where E_ϕ is the ϕ component of the electric field. Then it is easily found from Maxwell's equations that

$$\frac{d^2g}{dr^2} + \left(\frac{k_0^2}{M} - \frac{3}{4r^2}\right)g = 0. \qquad (7)$$

This is the normal form of the wave equation in the present case. Equation (7) is independent of K and contains no derivatives of the electrical parameters of the plasma. Whatever the profiles of M and K, (7) has the same form as for an unmagnetized medium of relative permittivity equal to $1/M$.

An equation describing H-parallel fields, equivalent to (3), has previously been treated by numerical methods [1]–[3]. In dealing with problems involving electromagnetic waves in stratified plasmas, analytical and numerical methods are both made use of. For example, the plasma is sometimes taken to consist of a number of homogeneous regions [5], [8] and formulas suitable for numerical computations then obtained. However, it may then be useful to replace one of the homogeneous regions by a continuously varying one in which an analytical expression for the fields is known [8].

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References

[1] J. Galejs, "Propagation of VLF waves below a curved and stratified anisotropic ionosphere," *J. Geophys. Res.*, vol. 69, pp. 3639–3650, September 1, 1964.
[2] C. Yeh and W. V. T. Rusch, "Effects of electron-density distribution on the backscattering from an anisotropic plasma cylinder," *Electronics Letters*, vol. 1, p. 25, March 1965.
[3] C. Yeh and W. V. T. Rusch, "Interaction of microwaves with an inhomogeneous and anisotropic plasma column," *J. Appl. Phys.*, vol. 36, pp. 2302–2306, July 1965.
[4] J. R. Wait, *Electromagnetic Waves in Stratified Media*. Oxford: Pergamon, 1962, pp. 230 and 227.
[5] J. R. Wait, "Transverse propagation of electromagnetic waves in a cylindrically stratified plasma," *Proc. 1963 Radio Physics Symp.*, J. Brandstatter, Ed., Holden-Day, 1964.
[6] R. Burman, "On transverse electromagnetic wave propagation in a stratified magnetoplasma," *Proc. IEEE (Correspondence)*, vol. 53, p. 1249, September 1965.
[7] R. Burman, "Some electromagnetic wave functions for propagation in cylindrically stratified media," *IEEE Transactions on Antennas and Propagation (Communications)*, vol. AP-13, pp. 646–647, July 1965.
[8] J. R. Wait, "Electromagnetic scattering from a radially inhomogeneous sphere," *Appl Sci. Res.*, vol. B-10, pp. 441–450, 1963–1964.

Measurement of Microwave Power in WR112 Waveguide (7.05 to 10.0 GHz)

The Radio Standards Laboratory of the NBS Institute for Basic Standards (U. S. Department of Commerce), Boulder, Colo., announces a calibration service for the measurement of effective efficiency and calibration factor of bolometer units and bolometer-coupler units in WR112 waveguide. Although calibrations can be performed within the useful range of the waveguide, some degree of economy to the customer results if calibrations are performed at the selected[1] frequencies of 7.75, 8.50, and 9 00 GHz.

The quantities measured in the new service are defined as follows:

Effective Efficiency for Bolometer Units: The ratio of the substituted dc power in the bolometer unit to the microwave power dissipated within the bolometer unit.

Calibration Factor for Bolometer Units: The ratio of the substituted dc power in the bolometer unit to the microwave power incident upon the bolometer unit.

Calibration Factor for Bolometer-Coupler Units: The ratio of the substituted dc power in the bolometer unit on the side arm of the directional coupler to the microwave power incident upon a nonreflecting load attached to the output port of the main arm.

In the interest of speeding up the availability of a service for the calibration of bolometer units and bolometer-coupler units in WR112 waveguide, the working standards in WR112 waveguide were calibrated indirectly by means of an existing microwave microcalorimeter that was designed specifically as a reference standard for power measurements in WR90 waveguide (8.20 to 12.4 GHz).[2] In making such measurements, a bolometer unit of WR90 waveguide, calibrated previously in the microcalorimeter at the WR112 frequencies, is used in combination with a bolometer unit of WR112 waveguide. Although a discontinuity exists in the waveguide between the two units (no transition being used) that would normally cause a reflection of considerable magnitude, a technique[3,4] is used that reduces the measurement error to a negligible amount.

The effective efficiency and calibration factor of bolometer units and the calibration factor of bolometer-coupler units are measured with an uncertainty no greater than one percent in WR112 waveguide. For these measurements, the element can be of the barretter or thermistor type, and of either 100- or 200-ohm resistance, operating at a bias current between 3.5 and 15 mA. The bolometer units should be of either the fixed-tuned or untuned broadband type. Power measurements can be made on bolometer units over a range of 0.1 to 10 mW.

Power measurements can be made on bolometer-coupler units in WR112 waveguide with coupling ratios from 3 to 20 dB. A bolometer unit of either the fixed-tuned or untuned broadband type must be permanently attached to the side arm of the coupler. The coupler should have a directivity no less than 40 dB, and a VSWR no greater than 1.05 for the input and output ports of the main arm of the coupler.

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[1] In performing microwave calibrations, a considerable amount of time is usually needed to prepare the system for a measurement operation. Much of this preparation is related to adjustment of the system to the frequency of operation selected for the calibration. Time and cost often can be reduced by minimizing the number of times the operating frequency of the calibration system must be readjusted.

[2] G. F. Engen, "A refined X-band microwave microcalorimeter," *NBS J. Res.*, vol. 63C, p. 77–82, July–September, 1959

[3] G. F. Engen, "A transfer instrument for the intercomparison of microwave power meters," *IRE Transactions on Instrumentation*, vol. I-9, pp. 202–208, September 1960.

[4] G. F. Engen, "A variable impedance power meter and adjustable reflection coefficient standard," *NBS J. Res.*, vol. 68C, p. 7–24, January-March, 1964.